For B.Sc. Part I 1st year in Trigonometry Part II in the chapter of Gregory series

S. R. Gupta's series for computing the value of π

A more convergent series than the preceding is the S. R. Gupta's Series, which is derived from the expression

$$3\ arctan\ \frac{1}{4} + arctan\ \frac{5}{99} = \frac{\pi}{4}$$

By substituting in succession $\frac{1}{4}$ and $\frac{5}{99}$ for x in

$$arctan\ x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots \infty$$

We have

$$\frac{\pi}{4} = 3\left[\frac{1}{4} - \frac{1}{3}.\frac{1}{4^3} + \frac{1}{5}.\frac{1}{4^5} - \frac{1}{7}.\frac{1}{4^7} + \cdots \infty\right] + \left[\frac{5}{99} - \frac{1}{3}.\frac{5^3}{99^3} + \frac{1}{5}.\frac{5^5}{99^5} - \frac{1}{7}.\frac{5^7}{99^7} + \cdots \infty\right]$$

$$\pi = 12\left[\frac{1}{4} - \frac{1}{3}.\frac{1}{4^3} + \frac{1}{5}.\frac{1}{4^5} - \frac{1}{7}.\frac{1}{4^7} + \cdots \infty\right] + 4\left[\frac{5}{99} - \frac{1}{3}.\frac{5^3}{99^3} + \frac{1}{5}.\frac{5^5}{99^5} - \frac{1}{7}.\frac{5^7}{99^7} + \cdots \infty\right]$$

now adding positive terms

Ist term of Ist series $\left(12.\frac{1}{4}\right) = 3.0000000000$

IIIrd term of Ist series $\left(12.\frac{1}{5}.\frac{1}{4^5}\right) = 0.0023437500$

Vth term of Ist series $\left(12.\frac{1}{9}.\frac{1}{4^9}\right) = 0.0000050862$

VIIth term of Ist series $\left(12.\frac{1}{13}.\frac{1}{4^{13}}\right) = 0.0000000137$

Positive term of IInd series

Ist term of IInd series $\left(4.\frac{5}{99}\right) = 0.2020202020$

IIIrd term of IInd series $\left(4.\frac{1}{5}.\frac{5^5}{99^5}\right) = 0.0000002628$

Total (+) 3.2043693147

Now adding Negative Terms

IInd term of Ist series $\left(12.\frac{1}{3}.\frac{1}{4^3}\right) = 0.0625$

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IV[th] term of I[st] series $\left(12.\frac{1}{7}.\frac{1}{4^7}\right) = 0.0001046316$

VI[th] term of I[st] series $\left(12.\frac{1}{11}.\frac{1}{4^{11}}\right) = 0.0000002600$

Negative Terms of II[nd] series

II[nd] term of II[nd] series $\left(4.\frac{1}{3}.\frac{5^3}{99^3}\right) = 0.0001717683$

Total (−) 0.0627766599

Hence ,

$$\begin{array}{r} (+)3.2043693147 \\ (-)0.0627766599 \\ \hline \pi = 3.14159265/48 \end{array}$$

This is the value of π correct to 8 places of decimals by taking the first series to 15 terms and the second series to 6 terms we should get π correct to 17 places of decimal.

In Machin series, by taking the first series to 21 terms and the second series to three terms we should get π correct to 16 places of decimal.

Therefore this series is simpler and converges faster than Machin series

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B.Sc.I in Trigonometry Part II Gregory series,
For class [redacted] in the chapter of Inverse circular Function

For B.Sc. I[st] year in Trigonometry Part II in the chapter Gregory series

Verification of S. R. Gupta's Formula for Computation of Pi

$$3\,arctan\frac{1}{4}+arctan\frac{5}{99} = \frac{\pi}{4}$$

$$3\,arctan\frac{1}{4}+arctan\frac{5}{99} = \text{arctan } 1$$

$$L.H.S. = 2\,arctan\frac{1}{4}+arctan\frac{1}{4}+arctan\frac{5}{99}$$

$$= arctan\left(\frac{2*\frac{1}{4}}{1-\frac{1}{16}}\right)+arctan\left(\frac{\frac{1}{4}+\frac{5}{99}}{\frac{1}{1}-\frac{1}{4}*\frac{5}{99}}\right)$$

$$= arctan\left(\frac{\frac{1}{2}}{\frac{15}{16}}\right)+arctan\left(\frac{\frac{119}{396}}{\frac{391}{396}}\right)$$

$$= arctan\left(\frac{8}{15}\right)+arctan\left(\frac{119}{391}\right)$$

$$= arctan\left(\frac{\frac{8}{15}+\frac{119}{391}}{1-\frac{8}{15}*\frac{119}{391}}\right)$$

$$= arctan\left(\frac{4913}{4913}\right)$$

$$= arctan\,1$$

$$L.H.S. = R.H.S.$$

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Modification of Machin Formula for Calculation of Pi

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ABSTRACT: In this paper I have Modified a Machin formula for calculation of the value of the Pi.

Introduction

in the year 1706 John Machin use the Gregory-Leibniz series to produce an algorithm that converged much faster.

$$\frac{\pi}{4} = 4\,arctan\frac{1}{5} - arctan\frac{1}{239}$$

Machin reached 100 digits of Pi with this formula. Machin formula remained the best known method for calculating Pi well into the age of computers.

History:- Chronology of Co€mputation of Pi

The fact that ratio of circumference to the diameter of a circle is a constant was known as Pi. Pi is an irrational number and it is also a transcendental number which implies that no finite sequence of algebraic operations on integers can render its value.

Polygon approximation Era: Greek mathematician Archimedes provide the first theoretical calculation of pi around 200BC. He said the constant takes the value between 223/71 and 22/7.

The Indian mathematician Aryabhata (476 - 550) made the approximation of Pi using a regular polygon of 384 sides and gave its value as 62832/20000 which is equal to 3.1416 and was correct up to 4 decimal places.

In the 5th century Chinese Mathematician Tsu Ch'ung used a variation of Archimedes method to give the value of Pi as 355/113 which is actually in the range between 3.1415926 and 3.1415927 this value of pi was correct up to 7 decimal places.

Infinite Series: During the year 1400 Indian mathematician Madhava used a series to calculate Pi. He used the following series.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots\ldots\ldots\ldots\ldots \infty$$

From this series he calculated the approximate value of Pi as 3.14159265359 which was correct up to 11 decimal places. This was a great achievement, since Europeans colleagues were still way behind this approximation during the same time.

In Europe, Madhava's formula was rediscovered by Scottish mathematician James Gregory in 1671 and by Leibniz in 1674

$$arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots \infty$$

When x=1, arc tan 1 = $\pi/4$ $\quad \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots\ldots\ldots\ldots\ldots \infty$

The Gregory - Leibniz series is simple but converges very slowly.

- In 1706 John Machin used the Gregory - Leibniz series to produce an algorithm that converged much faster

$$\frac{\pi}{4} = 4arctan\frac{1}{5} - arctan\frac{1}{239}$$

Ramanujan Era: Srinivasa Ramanujan discovered some new infinite series formula in 1910 but there importance was rediscovered around 1970 long after his death. One of his elegant formula was like this

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!.(1103 + 26390k)}{(k!)^4\, 396^{4k}}$$

Each addition of a term in Ramanujans series could give approximately additional 8 digits to Pi.

Solution of Modified formula

$$\frac{\pi}{4} = 3arctan\frac{1}{4} + arctan\frac{5}{99}$$

$$\pi = 12\left[\frac{1}{4} - \frac{1}{3}.\frac{1}{4^3} + \frac{1}{5}.\frac{1}{4^5} - \cdots \ldots\ldots\infty\right] + 4\left[\frac{5}{99} - \frac{1}{3}.\left(\frac{5^3}{99^3}\right) + \frac{1}{5}.\left(\frac{5^5}{99^5}\right) - \cdots \ldots\ldots\ldots\infty\right]$$

Result Analysis

By using computer program for *arctanx* we can calculate the value of Pi. By taking 15 terms of Ist series and six terms of IInd series, we get the value $\pi = 3.14159265358979323....$ upto 17 places of decimal accurately. If we take ~~180~~ 90 terms of Ist series and ~~72~~ 36 terms of IInd series we get accurate value of pi up to 95 places of decimal.

Conclusion

This modified formula is simpler than Machin formula and converges very fast. In Machin formula arctan $\frac{1}{239}$ is used which involves very large number of calculation where as we have used arctan $\frac{5}{99}$ which reduces the calculations.

References

Plane trigonometry part-II by S. L. Loney, Prof. of Mathematics at the Royal Holloway College (University of London) Sometime Fellow of Sidney Sussex College Cambridge